Three ways to write relationships for data are tables, words (descriptions), and rules. The pattern in tables between input (x) and output (y) values usually establishes the rule for a relationship. If you know the rule, it may be used to generate sets of input and output values. A description of a relationship may be translated into a table of values or a general rule (equation) that describes the relationship between the input values and output values. Each of these three forms of relationships may be used to create a graph to visually represent the relationship. For additional information, see the Math Notes boxes in Lessons 3.1.3, 3.1.4, 3.1.5, and 3.2.1 of the *Core Connections, Course 3* text.

Example 1

Complete the table by determining the relationship between the input (x) values and output (y) values, write the rule for the relationship, then graph the data.

input (x)	4	–3		5	0		3	–2	x
output (y)	8		4	10	0	–2		–4	

Begin by examining the four pairs of input values: 4 and 8, 5 and 10, 0 and 0, –2 and –4. Determine what arithmetic operation(s) are applied to the input value of each pair to get the second value. The operation(s) applied to the first value must be the *same* in all four cases to produce each given output value. In this example, the second value in each pair is twice the first value. Since the pattern works for all four points, make the conjecture that the rule is y (output) = $2x$ (input). See the completed table below for the missing values. The rule is $y = 2x$. Finally, graph each pair of data on an xy-coordinate system, as shown at right.
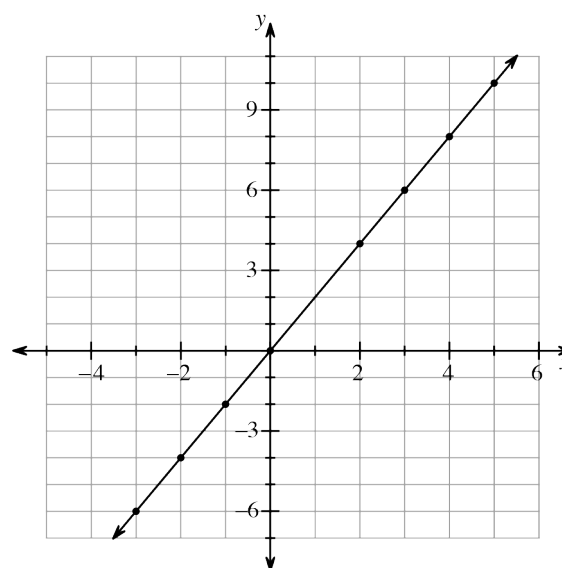
input (x)	4	–3	**2**	5	0	**–1**	3	–2	x
output (y)	8	**–6**	4	10	0	–2	**6**	–4	$y = 2x$

Example 2

Complete the table by determining the relationship between the input (x) and output (y) values, then write the rule for the relationship.

| input (x)	2	–1	4	–3	0	–2	1		x
output (y)	3	–3	7	–7	–1	–5	1		

Use the same approach as Example 1. In this table, the relationship is more complicated than simply multiplying the input value or adding (or subtracting) a number. Use a Guess and Check approach to try different patterns. For example, the first pair of values could be found by the rule $x + 1$, that is, $2 + 1 = 3$. However, that rule fails when you check it for –1 and –3: $-1 + 1 \neq -3$. From this guess you know that the rule must be some combination of multiplying the input value and then adding or subtracting to that product. The next guess could be to double x. Try it for the first two or three input values and see how close each result is to the known output values: for 2 and 3, $2(2) = 4$; for –1 and –3, $2(-1) = -2$; and for 4 and 7, $2(4) = 8$. Notice that each result is one more than the actual output value. If you subtract 1 from each product, the result is the expected output value. Make the conjecture that the rule is y (output) $= 2x$ (input) $- 1$ and test it for the other input values: for –3 and –7, $2(-3) - 1 = -7$; for 0 and –1, $2(0) - 1 = -1$; for –2 and –5, $2(-2) - 1 = -5$; and for 1 and 1, $2(1) - 1 = 1$. So the rule is $y = 2x - 1$.

Example 3

Complete the table below for $y = -2x + 1$, then graph each of the points in the table.

input (x)	–4	–3	–2	–1	0	1	2	3	4		x
output (y)											

Replace x with each input value, multiply by –2, then add 1. The results are ordered pairs: $(-4, 9)$, $(-3, 7)$, $(-2, 5)$, $(-1, 3)$, $(0, 1)$, $(1, -1)$, $(2, -3)$, $(3, -5)$, and $(4, -7)$. Plot these points on the graph (see Four-Quadrant Graphing if you need help with the fundamentals of graphing).

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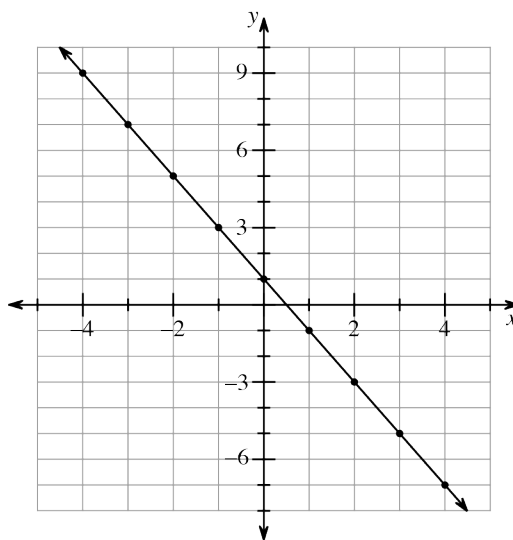
Example 4

Complete the table below for $y = x^2 - 2x + 1$, then graph the pairs of points and connect them with a smooth curve.

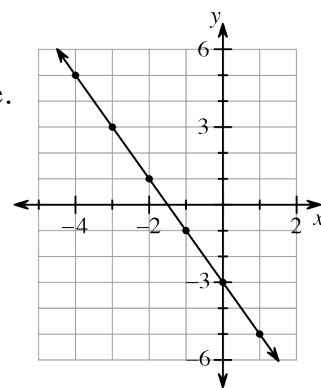| input (x)	–2	–1	0	1	2	3	4	x
output (y)								$x^2 - 2x + 1$

Replace x in the equation with each input value. Square the value, multiply the value by –2, then add both of these results and 1 to get the output (y) value for each input (x) value. The results are ordered pairs: $(-2, 9)$, $(-1, 4)$, $(0, 1)$, $(1, 0)$, $(2, 1)$, $(3, 4)$, and $(4, 9)$.
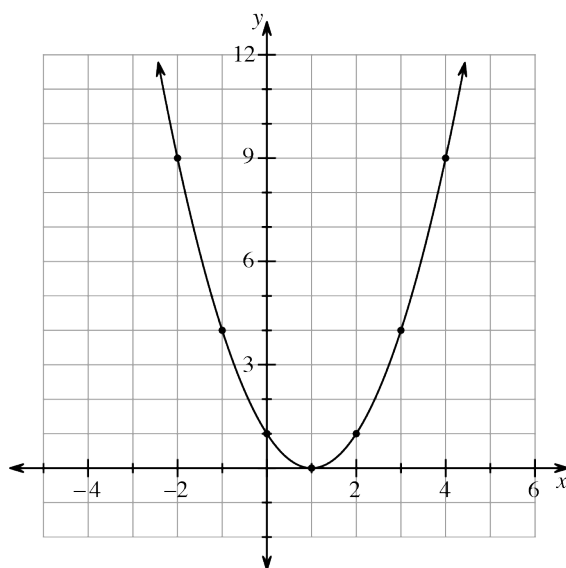
Example 5

Make an $x \to y$ table for the graph at right, then write a rule for the table.

input (x)	–4	–3	–2	–1	0	1	x
output (y)	5	3	1	–1	–3	–5	$-2x - 3$

Working left to right on the graph, read the coordinates of each point and record them in the table.

Guess and check by multiplying the input value, then adding or subtracting numbers to get the output value. For example you could start by multiplying the input value by 2: $2(-4) = -8$, $3(-4) = -12$, $2(-3) = -6$, etc. The results are not close to the correct output value. The product is also the opposite sign (+ –) of what you want. Your next choice could be to multiply by –2: $-2(-4) = 8$, $-2(-3) = 6$, $-2(-2) = 4$. Each result is three more than the expected output value, so make the conjecture that the rule is $y = -2x - 3$. Test it for the remaining points: $-2(-1) - 3 = -1$, $-2(0) - 3 = -3$, and $-2(1) - 3 = -5$. The rule is $y = -2x - 3$.

Problems

Complete each table. Then write a rule relating *x* and *y*.

1.

input (*x*)	10	5	20		–7	3	*x*
output (*y*)	14	9		–4	–3		

2.

input (*x*)	22	5	11		–9	12	*x*
output (*y*)	19	2		–8	–12		

3.

input (*x*)	10	0	15		–7	6	*x*
output (*y*)	25	–5		–8		13	

4.

input (*x*)	10	–3	4		–5	15	*x*
output (*y*)	–30	9	–12	–1			

5.

input (*x*)	–4	0	12		–16		*x*
output (*y*)	0	2		–4	–6	6	

6.

input (*x*)	4	–6		2	–12		*x*
output (*y*)	7	–3	6		–9	1	

7.

input (*x*)	3	0		–12	16	9	*x*
output (*y*)	3	–3	13		29		

8.

input (*x*)	2	0	–4		–13		*x*
output (*y*)	–5	1		–20	40	–17	

9.

input (*x*)	9	–6		0	3	–3	*x*
output (*y*)	3	–2	4		1		

10.

input (*x*)	–3	–2	–1	0	1	2	*x*
output (*y*)	–7	–4	–1	2	5	8	

11.

input (*x*)	–6	–4	–2	0	2	4	*x*
output (*y*)	1	2	3	4	5	6	

12.

input (*x*)	–2	–1	0	1	2	3	*x*
output (*y*)	5	2	1	2	5	10	

Complete a table for each rule, then graph and connect the points. For each rule, start with a table like the one below.

input (*x*)	–3	–2	–1	0	1	2	3
output (*y*)							

13. $y = 3x - 2$

14. $y = \frac{1}{2}x + 1$

15. $y = -x + 2$

16. $y = x^2 - 6$

     Core Connections, Course 3
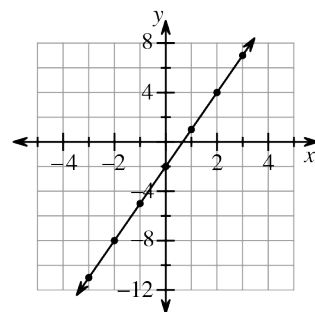
Answers

1. $24, -8, 7;\ y = x + 4$

2. $8, -5, 9;\ y = x - 3$

3. $40, -1, -26;\ y = 3x - 5$

4. $\frac{1}{3}, 15, -45;\ y = -3x$

5. $8, -12, 8;\ y = -\frac{1}{2}x + 2$

6. $3, 5, -2;\ y = x + 3$

7. $8, -27, 15;\ y = 2x - 3$

8. $13, 7, 6;\ y = -3x + 1$

9. $12, 0, -1;\ y = \frac{x}{3}$

10. $y = 3x + 2$

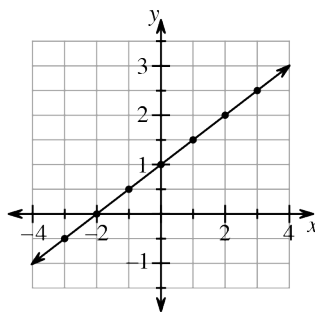11. $y = \frac{1}{2}x + 4$

12. $y = x^2 + 1$

13.

input (x)	–3	–2	–1	0	1	2	3
output (y)	–11	–8	–5	–2	1	4	7

14.

input (x)	–3	–2	–1	0	1	2	3
output (y)	–0.5	0	0.5	1	1.5	2	2.5

15.

input (x)	–3	–2	–1	0	1	2	3
output (y)	5	4	3	2	1	0	–1
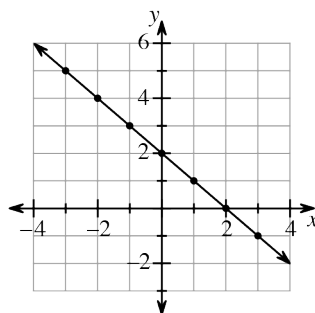
16.

input (x)	–3	–2	–1	0	1	2	3
output (y)	3	–2	–5	–6	–5	–2	3